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Mode-Matching Analysis of a Coaxially Fed Annular Slot Surrounded with Corrugations

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Abstract  Mode-matching analysis of a coaxially fed annular slot surrounded by corrugations is conducted in this study. An electromagnetic boundary value problem of a coaxially fed annular slot surrounded with corrugations is rigorously solved based on the Hankel transform, eigenfunction expansion, and mode-matching method. The radiated field is represented in a fast-convergent series that is amenable to numerical analysis. Numerical results are presented to illustrate the radiation behaviors in terms of the geometries of the corrugations and compared with measurement data.

Keywords  annular slot, Hankel transform, mode-matching method, surface plasmon polariton like modes, corrugations

1. Introduction

Recently, there have been extensive studies on beaming light from a subwavelength hole surrounded by surface corrugations related to the surface plasmon polariton (SPP) resonance theoretically, numerically, or experimentally (Lezec et al., 2002; Martín-Moreno et al., 2003; Thongrattanasiri et al., 2011). In fact, the SPP modes are the special TM surface wave modes in the optical region that are supported along a metal surface. Similarly, the SPP-like modes (spoof SPP or designer SPP) in the micro/millimeter wave regimes, which are bound electromagnetic (EM) surface waves mimicking the SPP modes in the optical region, can be sustained even by a perfect conductor, provided that its surface is periodically corrugated (Pendry et al., 2004; García-Vidal et al., 2005). Beaming from a subwavelength hole surrounded by corrugations of 1D or 2D version in the micro/millimeter wave regime due to the SPP-like modes has been also investigated theoretically, numerically, or experimentally (Hwang, 2011; Na et al., 2013; Lockyear et al., 2005; Caglayan et al., 2005). Previous studies have considered plane wave scattering from a subwavelength hole surrounded by corrugations in a conducting...
plane, but the directional radiation from a coaxially fed slot surrounded by corrugations was not presented.

Although there have been previous studies on the near-field plates or isoflux pattern antenna based on the concentric surface corrugations (Imani & Grbic, 2009, 2012; Jeon et al., 2011), these are not associated with the SPP-like modes. Also, previous theoretical works have considered the only fundamental mode (Martín-Moreno et al., 2003), but higher-order modes have not been considered. If the corrugation width increases, higher-order modes should be included in the EM analysis. Therefore, it is of interest to conduct mode-matching analysis of a coaxially fed annular slot surrounded with corrugations.

In this article, the boundary value problem dealing with a coaxially fed annular slot surrounded with corrugations will be solved by using the Hankel transform, eigenfunction expansion, and mode-matching method. The radiated fields are represented in a fast-convergent series and computed in terms of corrugation geometry to understand the radiation behaviors. Measurement is performed to check the validity of the formulations. Note that commercial full-wave EM tools can solve the present EM problem, but they cannot consider each eigenmode individually, and thus the effect of each mode on the scattering cannot be investigated. The proposed approach can consider the effects of each eigenmode on the radiation behaviors and can be used to optimize the corrugation geometries to obtain the minimum side-lobe level of radiation. Moreover, the presented analytical method is computationally more efficient than commercial full-wave EM tools.

2. Field Representations

Consider a coaxially fed cavity-backed annular slot surrounded by \( N \) concentric corrugations \((j = 1, 2, \ldots, N)\), as shown in Figure 1. Assume that an incident wave, TEM mode, propagates along the coaxial feed line. The dielectric constants of regions I, II, III, IV – \((j)\), and V are \( \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4^{(j)} \), and \( \epsilon_5 \), respectively. The wave numbers of regions I, II, III, and V are \( k_m = \omega \sqrt{\mu_0 \epsilon_m} (m = 1, 2, 3, 5) \) and region IV – \((j)\) is \( k_4^{(j)} = \omega \sqrt{\mu_0 \epsilon_4^{(j)}} \) \((j = 1, 2, \ldots, N)\), respectively. A time convention of \( e^{-i\omega t} \) is suppressed through the analysis. Only the TEM and TM\( _{0n} \) modes \((n = 1, 2, \ldots)\) in all regions are considered, since there is no variation of \( \phi \) \((\frac{\partial \phi}{\partial \theta} = 0; \) Kim & Eom, 2008\). In region I \((a < \rho < b, z < -g - h)\), the total EM field consists of the incident and reflected components. The incident and reflected components are

\[
\mathbf{E}_i (\rho, z) = e^{ik_1(z+g+h)} \mathbf{e}_l^{\text{tem}},
\]

\[
\mathbf{H}_i (\rho, z) = e^{ik_1(z+g+h)} \mathbf{h}_l^{\text{tem}},
\]

\[
\mathbf{E}_r (\rho, z) = A_0 e^{-ik_1(z+g+h)} \mathbf{e}_l^{\text{tem}} + \sum_{n_1=1}^{\infty} B_{0n_1} e^{-ik_{0n_1}(z+g+h)} \mathbf{e}_l^{n_1},
\]

\[
\mathbf{H}_r (\rho, z) = -A_0 e^{-ik_1(z+g+h)} \mathbf{h}_l^{\text{tem}} - \sum_{n_1=1}^{\infty} B_{0n_1} e^{-ik_{0n_1}(z+g+h)} \mathbf{h}_l^{n_1},
\]
where \( n_1 \) indicates the order of eigenmodes in region \( I \); \( \kappa_{0n_1} = \sqrt{k_1^2 - \gamma_{0n_1}^2} \) is determined by \( \Phi_0 (\gamma_{0n_1}^{(1)}, a, b) = 0 \):

\[
\overline{e}_{tem}^l = \hat{\rho} \frac{1}{\rho},
\]

(5)

\[
\overline{h}_{tem}^l = \hat{\phi} \frac{1}{\mu_0 \rho},
\]

(6)

\[
\overline{e}_{tm}^l = \hat{\rho} i \kappa_{0n_1}^{(1)} \gamma_{0n_1}^{(1)} \Phi_0^l (\gamma_{0n_1}^{(1)}, \rho, b),
\]

(7)

\[
\overline{h}_{tm}^l = \hat{\phi} i \gamma_{0n_1}^{(1)} \omega \varepsilon_1 \Phi_0^l (\gamma_{0n_1}^{(1)}, \rho, b),
\]

(8)

\[
\Phi_0 (\alpha \rho, \beta) = J_0 (\alpha \rho) - \frac{J_0 (\alpha \beta)}{N_0 (\alpha \beta)} N_0 (\alpha \rho).
\]

(9)
Note that $J_0(\cdot)$ and $N_0(\cdot)$ are the Bessel function of the first and second kind and order 0, respectively; $J'_0(\cdot)$, $N'_0(\cdot)$, and $\Phi'_0(\cdot)$ denote differentiation with respect to the argument, respectively. In region II ($a < \rho < c$, $-g - h < z < -g$), the EM field is given by

$$\vec{E}''_t(\rho, z) = (C_0 e^{ik_3(z+g)} + D_0 e^{-ik_3(z+g)})\vec{e}_\text{tem}$$

$$+ \sum_{n_2=1}^{\infty} \left( E_{n_2} e^{ik_{0n_2}(z+g)} + F_{n_2} e^{-ik_{0n_2}(z+g)} \right)\vec{e}_{tm},$$  

(10)

$$\vec{H}''_t(\rho, z) = (C_0 e^{ik_3(z+g)} - D_0 e^{-ik_3(z+g)})\vec{h}_\text{tem}$$

$$+ \sum_{n_2=1}^{\infty} \left( E_{n_2} e^{ik_{0n_2}(z+g)} - F_{n_2} e^{-ik_{0n_2}(z+g)} \right)\vec{h}_{tm},$$  

(11)

where $n_2$ indicates the order of eigenmodes in region II; $k_{0n_2} = \sqrt{k_2^2 - \gamma_{0n_2}^2}$ is determined by $\Phi_0(\gamma_{0n_2}^2 a, c) = 0$:

$$\vec{e}_\text{tem} = \hat{\rho} \frac{1}{\rho},$$

(12)

$$\vec{h}_\text{tem} = \hat{\phi} \sqrt{\frac{\varepsilon_2 - 1}{\mu_0 \rho}},$$

(13)

$$\vec{e}_{tm} = \hat{\mu} k_{0n_2}^2 \Phi_0^2(\gamma_{0n_2}^2 \rho, c),$$

(14)

$$\vec{h}_{tm} = \hat{\phi} i \gamma_{0n_2}^2 \omega_2 \Phi_0^2(\gamma_{0n_2}^2 \rho, c).$$

(15)

In region III ($a_0 < \rho < b_0$, $-g < z < 0$), the EM field takes the form of

$$\vec{E}'''_t(\rho, z) = (G_0 e^{ik_3(z+g)} + H_0 e^{-ik_3(z+g)})\vec{e}_\text{tem}$$

$$+ \sum_{n_3=1}^{\infty} \left( I_{n_3} e^{ik_{0n_3}(z+g)} + J_{n_3} e^{-ik_{0n_3}(z+g)} \right)\vec{e}_{tm},$$  

(16)

$$\vec{H}'''_t(\rho, z) = (G_0 e^{ik_3(z+g)} - H_0 e^{-ik_3(z+g)})\vec{h}_\text{tem}$$

$$+ \sum_{n_3=1}^{\infty} \left( I_{n_3} e^{ik_{0n_3}(z+g)} - J_{n_3} e^{-ik_{0n_3}(z+g)} \right)\vec{h}_{tm},$$  

(17)
where \( n_3 \) indicates the order of eigenmodes in region III; \( \kappa_{0n_3} = \sqrt{k_3^2 - \gamma_{0n_3}^2} \) is determined by \( \Phi_0(\gamma_{0n_3}^{(3)} a_0, b_0) = 0 \):

\[
E_{\text{tem}}^{\text{III}} = \hat{\rho} \frac{1}{\rho},
\]

\[
H_{\text{tem}}^{\text{III}} = \hat{\phi} \frac{1}{\mu_0 \rho},
\]

\[
E_{\text{tem}}^{\text{III}} = \hat{\rho} i \kappa_{0n_3}^{(3)} \gamma_{0n_3}^{(3)} \Phi_0 \left( \gamma_{0n_3}^{(3)} \rho, b_0 \right),
\]

\[
H_{\text{tem}}^{\text{III}} = \hat{\phi} i \gamma_{0n_3}^{(3)} \omega \epsilon_3 \Phi_0 \left( \gamma_{0n_3}^{(3)} \rho, b_0 \right).
\]

In region IV \((a_j < \rho < b_j, -d_j < z < 0)\), the EM field is

\[
E_{\text{em}}^{\text{IV}}(\rho, z) = i K_0^{(j)} \sin[k_4(z + d_j)] e_{\text{tem}}^{\text{IV}}(z) + \sum_{n_4=1}^{\infty} L_{0n_4}^{(j)} \sin \left[ \kappa_{0n_4}^{(4)(j)} (z + d_j) \right] e_{\text{tm}}^{\text{IV}}(z),
\]

\[
H_{\text{em}}^{\text{IV}}(\rho, z) = K_0^{(j)} \cos[k_4(z + d_j)] h_{\text{tem}}^{\text{IV}}(z) + \sum_{n_4=1}^{\infty} L_{0n_4}^{(j)} \cos \left[ \kappa_{0n_4}^{(4)(j)} (z + d_j) \right] h_{\text{tm}}^{\text{IV}}(z),
\]

where \( n_4 \) indicates the order of eigenmodes in region IV \((a_j < \rho < b_j, -d_j < z < 0)\); \( \kappa_{0n_4} = \sqrt{k_4^{(j)} - \gamma_{0n_4}^{(4)(j)} \gamma_{0n_4}^{(4)(j)}} \) is determined by \( \Phi_0(\gamma_{0n_4}^{(4)(j)} a_j, b_j) = 0 \):

\[
e_{\text{tem}}^{\text{IV}} = \hat{\rho} \frac{1}{\rho},
\]

\[
h_{\text{tem}}^{\text{IV}} = \hat{\phi} \frac{1}{\mu_0 \rho},
\]

\[
e_{\text{tm}}^{\text{IV}} = \hat{\rho} i K_0^{(4)(j)} \gamma_{0n_4}^{(4)(j)} \Phi_0 \left( \gamma_{0n_4}^{(4)(j)} \rho, b_j \right),
\]

\[
h_{\text{tm}}^{\text{IV}} = \hat{\phi} i \gamma_{0n_4}^{(4)(j)} \omega \epsilon_4 \Phi_0 \left( \gamma_{0n_4}^{(4)(j)} \rho, b_j \right).
\]

In region V \((z > 0)\), the EM field is expressed based on the Hankel transform as

\[
E_{\text{em}}^{\text{V}}(\rho, z) = \int_0^{\infty} A_\zeta(\zeta) E_{\text{em}}^{\text{V}} e^{ik_5 z} d\zeta,
\]

\[
H_{\text{em}}^{\text{V}}(\rho, z) = \int_0^{\infty} A_\zeta(\zeta) H_{\text{em}}^{\text{V}} e^{ik_5 z} d\zeta.
\]
where \( \kappa_5 = \sqrt{k_5^2 - \xi^2} \),

\[
\mathcal{E}_{tm}^{V} = \hat{\rho} i \kappa_5 \zeta J_0(\zeta \rho),
\]

\( \mathcal{H}_{tm}^{V} = \hat{\phi} i \omega \varepsilon_5 J_0(\zeta \rho). \)

3. Enforcement of Boundary Conditions

It is necessary to enforce the field continuities. The tangential electric field continuities at \( z = -g - h \) require

\[
\mathcal{E}_t^H(\rho, -g - h) = \begin{cases} 
\mathcal{E}_t(\rho, -g - h) + \mathcal{E}_t(\rho, -g - h) & a < \rho < b, \\
0 & \text{elsewhere}.
\end{cases}
\]  

(32)

For further simplification, it is convenient to use the power orthogonality, which is summarized in Appendix A. Applying the power orthogonality properties to Eq. (32) gives

\[
(C_0 + D_0) \ln \frac{a}{c} = (1 + A_0) \ln \frac{a}{b},
\]

(33)

\[
(E_{0q_2} + F_{0q_2}) \left[ \frac{2k_{0q_2}^{(2)}}{N_0^2 \gamma_{0q_2}^{(2)}} \right]^a_{z=c} = -i(1 + A_0)\pi^2 \Phi_0 \left( \gamma_{0q_2}^{(2)} b, c \right) + \sum_{n_1=1}^{\infty} B_{0n_1} \frac{2\pi k_{0n_1}^{(1)} \gamma_{0q_2}^{(2)}}{\left( \gamma_{0n_1}^{(1)} - \gamma_{0q_2}^{(2)} \right)} \Phi_0 \left( \gamma_{0n_1}^{(1)} b, c \right),
\]

(34)

where \( f(z) \big|_{z=A}^{B} = f(B) - f(A) \). The continuity of the tangential magnetic field on the coaxial feed line (region I) and \( z = -g - h \) is

\[
\mathcal{B}_t^I(\rho, -g - h) + \mathcal{B}_t^I(\rho, -g - h) = \mathcal{B}_t^H(\rho, -g - h), \quad a < \rho < b.
\]

(35)

Applying the power orthogonality properties to Eq. (35) yields

\[
(1 - A_0) \sqrt{\frac{\varepsilon_1}{\mu_0}} \ln \frac{a}{b} = (C_0 - D_0) \sqrt{\frac{\varepsilon_2}{\mu_0}} \ln \frac{a}{b} + i \sum_{n_2=1}^{\infty} (E_{0n_2} - F_{0n_2}) \omega \varepsilon_2 \Phi_0 \left( \gamma_{0n_2}^{(2)} b, c \right),
\]

(36)

\[
B_{0q_1} \xi_1 \left[ \frac{1}{N_0^2 \gamma_{0q_1}^{(2)}} \right]^b_{z=a} = \sum_{n_2=1}^{\infty} (E_{0n_2} - F_{0n_2}) \frac{\pi \varepsilon_2 \gamma_{0n_2}^{(2)}}{\left( \gamma_{0n_1}^{(1)} - \gamma_{0q_2}^{(2)} \right)} \Phi_0 \left( \gamma_{0n_2}^{(2)} b, c \right). \]

(37)
The tangential electric field continuities at $z = -g$ require

$$E^H_i (\rho, -g) = \begin{cases} E^H_i (\rho, -g), & a_0 < \rho < b_0 \\ 0 & \text{elsewhere.} \end{cases} \quad (38)$$

Applying the power orthogonality properties to Eq. (38) yields

$$E_{0q_2} e^{ik_{0q_2} z} + F_{0q_2} e^{-ik_{0q_2} z} \left[ \frac{2k^{(2)}_{0q_2}}{N_0^2 (\gamma_{0q_2} z)} \right]^{c}_{z=a}$$

$$= -i (G_0 + H_0) \pi^2 \left[ \Phi_0 \left( \gamma^{(3)}_{0q_3} z, c \right) \right]^{b_0}_{z=a_0}$$

$$- \sum_{n_3=1}^{\infty} \left( I_{0n_3} + J_{0n_3} \right) 2\pi \kappa_{0n_3}^{(3)} \gamma_{0q_3}^{(2)} J_0 \left( \gamma_{0n_3}^{(1)} b_0 \right) \left[ \Phi_0 \left( \gamma^{(3)}_{0q_3} z, c \right) \right]^{b_0}_{z=a_0}. \quad (40)$$

The continuity of the tangential magnetic field on the annular slot (region III) and $z = -g$ is

$$\mathbf{\Pi}^H_i (\rho, -g) = \mathbf{\Pi}^H_i (\rho, -g), \quad a_0 < \rho < b_0. \quad (41)$$

Applying the power orthogonality properties to Eq. (41) gives

$$(G_0 - H_0) \sqrt{\frac{\varepsilon_3}{\mu_0}} \ln \frac{a_0}{b_0} = (C_0 e^{ik_2 h} - D_0 e^{-ik_2 h}) \sqrt{\frac{\varepsilon_2}{\mu_0}} \ln \frac{a_0}{b_0}$$

$$- i \sum_{n_2=1}^{\infty} \left( E_{0n_2} e^{ik_{0n_2} h} - F_{0n_2} e^{-ik_{0n_2} h} \right) \omega \varepsilon_2 \left[ \Phi_0 \left( \gamma^{(2)}_{0q_2} z, c \right) \right]^{b_0}_{z=a_0}, \quad (42)$$

$$- (I_{0q_3} - J_{0q_3}) \varepsilon_3 \left[ \frac{1}{N_0^2 (\gamma_{0q_3} z)} \right]^{b_0}_{z=a_0} = \sum_{n_2=1}^{\infty} \left( E_{0n_2} e^{ik_{0n_2} h} - F_{0n_2} e^{-ik_{0n_2} h} \right)$$

$$\times \left[ \frac{\pi \varepsilon_2 \gamma^{(2)}_{0q_2} J_0 \left( \gamma^{(3)}_{0n_2} b_0 \right) \left[ \Phi_0 \left( \gamma^{(3)}_{0q_3} z, c \right) \right]}{N_0 \left( \gamma^{(3)}_{0q_3} b_0 \right) \gamma^{(2)}_{0q_2} N_0 \left( \gamma^{(3)}_{0n_2} \right)} \right]^{a_0}_{z=b_0}. \quad (43)$$

The tangential electric field continuities at $z = 0$ require

$$E^V_i (\rho, 0) = \begin{cases} E^H_i (\rho, 0), & a_0 < \rho < b_0 \\ E^{IV-(j)}_i (\rho, 0), & a_j < \rho < b_j, \\ 0 & \text{elsewhere.} \end{cases} \quad (44)$$
Applying the power orthogonality to Eq. (44) yields

\[ \tilde{A}^V_2(\zeta) = -i(G_0e^{ik_3g} - H_0e^{-ik_3g})[J_0(\zeta)]_{z=a_0}^b_0 \frac{1}{k_5 \zeta} \]

\[ - \sum_{n_3=1}^{\infty} \left( I_{0n_3}e^{ik_{0n_3}g} + J_{0n_3}e^{-ik_{0n_3}g} \right) \kappa_{0n_3}^{(3)} \frac{2}{\pi} \left[ \frac{J_0(\zeta)}{N_0(\gamma_{0n_3}^3 \zeta)} \right]_{z=a_0}^b_0 \]

\[ + \sum_{j=1}^{N} K_0^{(j)} \sin \left( k_4^{(j)} d_j \right) [J_0(\zeta)]_{z=a_j}^b_j \frac{1}{k_5 \zeta} \]

\[ - i \sum_{j=1}^{N} \sum_{n_4=1}^{\infty} L_{0n_4}^{(j)} \sin \left( \kappa_{0n_4}^{(4)(j)} d_j \right) \kappa_{0n_4}^{(4)(j)} \frac{2}{\pi} \left[ \frac{J_0(\zeta)}{N_0(\gamma_{0n_4}^{(4)(j)} \zeta)} \right]_{z=a_j}^b_j . \] \tag{45} \]

The continuity of the tangential magnetic field on the annular slot and \( z = 0 \) is

\[ \overline{H}^V_1(\rho, 0) = \overline{H}^V_1(\rho, 0), \quad a_0 < \rho < b_0. \] \tag{46} \]

Applying the power orthogonality properties to Eq. (46), substituting \( \tilde{A}^V_2(\zeta) \) and carrying out a lengthy algebraic manipulation gives

\[ (G_0e^{ik_3g} - H_0e^{-ik_3g}) \sqrt{\frac{\varepsilon_3}{\mu_0}} \ln \frac{a_0}{b_0} \]

\[ = -(G_0e^{ik_3g} + H_0e^{-ik_3g})\omega \varepsilon_5 I_1 - i \sum_{n_3=1}^{\infty} \left[ I_{0n_3}e^{ik_{0n_3}g} + J_{0n_3}e^{-ik_{0n_3}g} \right] \frac{2\omega \varepsilon_5 \kappa_{0n_3}^{(3)}}{\pi} I_2 \]

\[ - i \sum_{j=1}^{N} K_0^{(j)} \sin \left( k_4^{(j)} d_j \right) \omega \varepsilon_5 I_3 + \sum_{j=1}^{N} \sum_{n_4=1}^{\infty} L_{0n_4}^{(j)} \sin \left( \kappa_{0n_4}^{(4)(j)} d_j \right) \frac{2\omega \varepsilon_5 \kappa_{0n_4}^{(4)(j)}}{\pi} I_4. \] \tag{47} \]

\[ - \left( I_{0q_3}e^{ik_{0q_3}g} - J_{0q_3}e^{-ik_{0q_3}g} \right) \varepsilon_3 \left[ \frac{1}{N_0^2(\gamma_{0q_3}^3 a_0)} \right]_{z=a_0}^b_0 \]

\[ = i \left( G_0e^{ik_3g} + H_0e^{-ik_3g} \right) \pi \varepsilon_5 I_5 - \sum_{n_3=1}^{\infty} \left[ I_{0n_3}e^{ik_{0n_3}g} + J_{0n_3}e^{-ik_{0n_3}g} \right] \kappa_{0n_3}^{(3)} 2\varepsilon_5 I_6 \]

\[ - \sum_{j=1}^{N} K_0^{(j)} \sin \left( k_4^{(j)} d_j \right) \pi \varepsilon_5 I_7 - i \sum_{j=1}^{N} \sum_{n_4=1}^{\infty} L_{0n_4}^{(j)} \sin \left( \kappa_{0n_4}^{(4)(j)} d_j \right) \kappa_{0n_4}^{(4)(j)} 2\varepsilon_5 I_8. \] \tag{48} \]
The continuity of the tangential magnetic field on the \((t)\)th concentric corrugation (region \(IV - (t)\)) and \(z = 0\) is

\[
\mathbf{\Pi}^V_{(t)\to(0)}(\rho,0) = \mathbf{\Pi}^V(\rho,0), \quad a_t < \rho < b_t.
\]  

(49)

Applying the power orthogonality properties to Eq. (49), substituting \(A_2^V(\xi)\), and carrying out a lengthy algebraic manipulation gives

\[
K_0^{(t)} \cos\left(k_4^{(t)} d_t\right) \sqrt{\frac{\epsilon_4^{(t)}}{\mu_0}} \ln \frac{a_t}{b_t}
\]

\[
= -(G_0 e^{ik_3 g} + H_0 e^{-ik_3 g}) \omega \varepsilon k I_9 - i \sum_{n_3 = 1}^{\infty} \left(I_{0n_3} e^{ik_{0n_3} g} + J_{0n_3} e^{-ik_{0n_3} g}\right) \frac{2 \omega \varepsilon k_{0n_2}^{(3)}}{\pi} l_{10}
\]

\[
- i \sum_{j = 1}^{N} K_0^{(j)} \sin\left(k_4^{(j)} d_j\right) \omega \varepsilon k I_{11} + \sum_{j = 1}^{N} \sum_{n_4 = 1}^{\infty} L_{0n_4}^{(j)} \sin\left(k_{0n_4}^{(j)} d_j\right) \frac{2 \omega \varepsilon k_{0n_4}^{(4)(j)}}{\pi} l_{12},
\]  

(50)

\[
L_{0q_4}^{(t)} \cos\left(k_{0q_4}^{(4)(t)} d_t\right) \epsilon_4^{(t)} \left[\frac{1}{N_0^2 (\gamma_{0q_4}^{(4)(t)})^2}\right]_{z = a_t}
\]

\[
= -i(G_0 e^{ik_3 g} + H_0 e^{-ik_3 g}) \pi \varepsilon k I_{13} + \sum_{n_3 = 1}^{\infty} \left(I_{0n_3} e^{ik_{0n_3} g} + J_{0n_3} e^{-ik_{0n_3} g}\right) \kappa_{0n_3}^{(3)} \varepsilon k I_{14}
\]

\[
+ \sum_{j = 1}^{N} K_0^{(j)} \sin\left(k_4^{(j)} d_j\right) \pi \varepsilon k I_{15} + i \sum_{j = 1}^{N} \sum_{n_4 = 1}^{\infty} L_{0n_4}^{(j)} \sin\left(k_{0n_4}^{(j)} d_j\right) \kappa_{0n_4}^{(4)(j)} \varepsilon k I_{16},
\]  

(51)

where the integrals in Eqs. (47), (48), (50), and (51) are given in Appendix B. It is possible to solve a set of simultaneous equations (Eqs. (33), (34), (36), (37), (39), (40), (42), (43), (47), (48), (50), and (51)) for the discrete modal coefficients \(A_0, B_{0n_1}, C_0, D_0, E_{0n_2}, F_{0n_2}, G_0, H_0, I_{0n_3}, J_{0n_3}, K_0^{(j)},\) and \(L_{0n_4}^{(j)} (j = 1, 2, \ldots, N).\)

4. Numerical Results and Experiments

The radiated field in a far zone can be calculated based on the stationary phase approximation as

\[
H_\phi^V \approx \tilde{A}_2^V(k_5 \sin \theta) \omega \varepsilon k R \cos \frac{e^{ik_5 r}}{r},
\]  

(52)
where \((r, \theta)\) indicates the observed point in a far-zone and

\[
A_z^V(k_5 \sin \theta) = -i(G_0 e^{ik_3 R} + H_0 e^{-ik_3 R}) \left[ \frac{J_0(k_5 \sin \theta z)}{k_5 \cos \theta \sin \theta} \right]_{z=a_0}^{b_0} \]

\[
- \sum_{m=1}^{\infty} \left( I_{0n3} e^{i\kappa_{0n3} R} + J_{0n3} e^{-i\kappa_{0n3} R} \right) \kappa_{0n3}^2 \frac{J_0(k_5 \sin \theta z)}{N_0 \left( \gamma_{0n3}^3 z \right)} \bigg|_{z=a_0}^{b_0} \]

\[
+ \sum_{j=1}^{N} \kappa_0^{(j)} \sin \left( k_4 (j) d_j \right) \frac{J_0(k_5 \sin \theta z)}{k_5 \cos \theta \sin \theta} \bigg|_{z=a_0}^{b_j} \]

\[
- i \sum_{j=1}^{N} \sum_{n=4}^{\infty} L^{(j)}_{0n4} \sin \left( \kappa_{0n4}^{(4)(j)} d_j \right) \kappa_{0n4}^{(4)(j)} \frac{J_0(k_5 \sin \theta z)}{N_0 \left( \gamma_{0n4}^{(4)(j)} z \right)} \bigg|_{z=a_0}^{b_j} + . (53)
\]

To check the validity of the proposed formulation, the coaxially fed annular slot surrounded with concentric corrugations is considered, as shown in Figure 2. The fabricated coaxially fed annular slot surrounded with concentric corrugations consists of an annular slot surrounded with concentric corrugations, a circular cylindrical cavity, a coaxial cable, and an subminiature version A (SMA) connector. The coaxial cable is a 50-ohm UT-250 semi-rigid cable (MICRO-COAX, USA) \((a = 0.815 \text{ mm}, b = 2.655 \text{ mm}, \epsilon_1 \approx 2.1 \epsilon_0)\), and its inner conductor is extended and connected to flange. Note that the design parameters of the cavity and annular slot are determined by \(c = 39.2 \text{ mm}, \epsilon_2 = \epsilon_0, a_0 = 6 \text{ mm}, b_0 = 8 \text{ mm}, \) and \(\epsilon_3 \approx 2.1 \epsilon_0\) to operate in the Ku-band frequency range.

Figure 3 illustrates the reflection coefficient versus frequency to check the impedance-matching characteristic of the fabricated coaxially fed annular slot with corrugations. The comparison between this study’s result and measurement data generally shows good agreement. It is seen that the case with the cavity has good matching characteristics with the 300-MHz bandwidth and resonant frequency around 14.8 GHz compared with the case without the cavity. It is assumed that each truncation numbers of four eigenmodes in regions \(I, II, III,\) and \(IV - (j)\), except for the fundamental mode (TEM mode), are \(M_1, M_2, M_3,\) and \(M_4\) due to different dimensions of each regions.

Table 1 shows the convergence behaviors of the modal coefficients of the case with the cavity in Figure 3. The truncation numbers of each eigenmode are set as \(M_1 = 3, M_2 = 15, M_3 = 5,\) and \(M_4 = 5\) to achieve convergence to within 0.5%. These truncation numbers are used through numerical examples unless specified otherwise.

Figure 4 shows the normalized radiation intensity versus elevation angle \(\theta\) for different numbers of corrugations. It is observed that the present result \((N = 6)\) agrees well with the measurement data. Note that the main lobe becomes narrower as the number of corrugations \(N\) increases. The gain enhancement of 15.33 dB is obtained employing six corrugations surrounding an annular slot compared with the case without corrugations. This beaming is due to the excitation of the SPP-like modes propagating along the corrugated surface and the in-phase radiation (Martín-Moreno et al., 2003) from all corrugations surrounding the annular slot. The computation time used in the case of six corrugations in Figure 4 is 342 sec on a PC (Intel Core CPU 760 i5 with 2.80 GHz and 4.0 GB of memory; Samsung, Korea), and the simulation time of Microwave Studio
(MWS) of computer simulation technology (CST) on a workstation (Intel Xeon CPU E5430 with 2.66 GHz and 24.0 GB of memory; Dell, USA) is 6,923 sec, implying that the proposed method is computationally more efficient.

To verify the existence of the SPP-like modes excited by corrugations, the time-averaged $\rho$ component of Poynting vector, $S_{av-\rho}(\rho, z)$, is plotted near the conductor surface at $f = 14.8$ GHz for the cases of an annular slot with corrugations in Figure 5. The comparison between the obtained result and the result of MWS of CST generally shows good agreement. It is interesting to note that the time-averaged Poynting vector is positive and larger than that of the no-corrgulation case due to the SPP-like modes. This means the SPP-like modes propagate along the $+\rho$-axis, thereby transporting EM energy to corrugations surrounding the annular slot.

Figure 2. Fabricated coaxially fed annular slot surrounded with concentric corrugations: (a) top view and (b) bottom view.
Figure 3. Reflection coefficient versus frequency ($w_j = 2$ mm, $d_j = 3$ mm, $T_j = 18.5$ mm, $N = 6$).

Figure 6 shows the side-lobe level of radiation pattern in terms of the width and the depth of corrugations to find the optimal conditions for the minimum side-lobe level of radiation pattern. Note that the corrugation width and depth have optimum values that make the radiation pattern directional. For example, the minimum side-lobe level of radiation pattern occurs when the corrugation width (9 mm) is around the half of

Table 1

Convergence behaviors of modal coefficients in the case ($N = 6$) of Figure 4

| TEM  | $|A_0|$   | $|C_0|$   | $|D_0|$   | $|B_{0n_1}|$ | $|E_{0n_2}|$ | $|F_{0n_2}|$ |
|------|----------|----------|----------|----------------|----------------|----------------|
| $n_1 = 1$ | $6.80e-5$ | $1.06e-3$ | $6.42e-4$ | $2.51e-4$ | $1.70e-4$ | $7.06e-7$ |
| $n_1 = 2$ | $1.98e-5$ | $2.27e-5$ | $7.06e-7$ | $2.12e-6$ | $4.37e-9$ |          |
| $n_1 = 3$ | $1.57e-6$ | $2.76e-6$ | $3.57e-12$ | $1.38e-11$ | $1.10e-6$ |          |
| $n_1 = 5$ | $1.90e-7$ | $3.75e-24$ |           | $1.93e-19$ | $3.02e-16$ |          |

| TEM  | $|A_0|$   | $|H_0|$   | $|K_{0}^{(1)}|$ | $|K_{0}^{(6)}|$ | $|B_{0n_1}|$ | $|J_{0n_3}|$ | $|L_{0n_3}^{(1)}|$ | $|L_{0n_3}^{(6)}|$ |
|------|----------|----------|----------------|----------------|----------------|----------|----------------|----------------|
| $n_3 = 1$ | $2.39$ | $1.43$ | $6.45$ | $4.32$ | $2.55e-8$ | $2.35e-16$ | $1.38e-11$ | $3.57e-12$ |
| $n_3 = 3$ | $2.76e-6$ | $3.75e-24$ |           | $1.93e-19$ | $3.02e-16$ |          |          |          |
| $n_3 = 5$ | $1.90e-7$ | $3.75e-24$ |           | $1.93e-19$ | $3.02e-16$ |          |          |          |
Figure 4. Normalized radiation intensity versus elevation angle $\theta$ ($w_j = 2 \text{ mm}, d_j = 3 \text{ mm}, T_j = 18.5 \text{ mm}, f = 15 \text{ GHz}$).

the corrugation period and the depth-to-width ratio is around 0.25. Also, the minimum side-lobe level is obtained when the width is very small and the depth is around $\lambda_{SPP}/4$.

If the corrugation width increases, higher-order modes must be included in the EM field analysis, and it is of great importance to investigate the effect of higher-order modes on the radiation behaviors. Toward this purpose, the normalized radiation intensity versus elevation angle $\theta$ for the wide corrugation is plotted (i) by considering only the fundamental mode (TEM mode) and (ii) by considering the fundamental mode plus higher-order modes (TEM mode and $M_4 = 5$) in Figure 7. It is seen that the case of $M_4 = 0$ (only the TEM mode) deviates from the measurement data around the main lobe of the radiation pattern, whereas the cases considering the TEM mode plus a few higher-order modes ($M_4 = 5$) agree well with the measurement data. This is because

Figure 5. Time averaged Poynting vector at $z = 1 \text{ mm}$ versus $\rho$ ($w_j = 2 \text{ mm}, d_j = 3 \text{ mm}, T_j = 18.5 \text{ mm}, f = 14.8 \text{ GHz}$).
Figure 6. Contour plot of the directivity versus corrugation width $w_j$ and depth $d_j$ ($T_j = 18.5$ mm, $N = 6$, $f = 15$ GHz).

Figure 7. Normalized radiation intensity versus elevation angle $\theta$ ($w_j = 12$ mm, $d_j = 3.5$ mm, $T_j = 18.5$ mm, $N = 6$, $f = 15$ GHz).
Figure 8. Normalized radiation intensity versus elevation angle $\theta$ ($w_j = 2$ mm, $d_j = 3$ mm, $N = 6$, $f = 15$ GHz).

The excitation of the first higher-order mode TM$_{01}$ becomes a propagating mode around $w_j = 10$ mm at $f = 15$ GHz in corrugations. Therefore, the effect of higher-order modes should not be neglected as the corrugation width increases.

To verify the beam steering, the different spacings of corrugations are considered in Figure 8 as

Case A: $T_j = \begin{cases} \Lambda/2 & j = \text{even}, \\ \Lambda & j = \text{odd}; \end{cases}$

Case B: $T_j = \begin{cases} \Lambda & j = \text{even}, \\ \Lambda/2 & j = \text{odd}, \end{cases}$

where $\Lambda = 18.5$ mm and $j = 0, \ldots, N - 1$. It is seen that Cases A and B have steering angles $\theta = 18.5^\circ$ and $22.5^\circ$, half-power beam widths (HPBWs) $8.5^\circ$ and $6.5^\circ$, and side-lobe levels $-8.1$ and $-5.4$ dB, respectively, whereas the standard case has a steering angle $\theta = 4.4^\circ$, HPBW $5.3^\circ$, and side-lobe level $-16.6$ dB, respectively. $T_j = \frac{\Lambda}{2}$ leads to the phase shift of the electric fields on corrugations, thereby making the out-of-phase radiation from each corrugation. Note that this phenomenon can find practical application in the beam steering antenna systems.

5. Conclusion

The EM boundary-value problem of a coaxially fed annular slot surrounded with corrugations has been solved based on the Hankel transform, eigenfunction expansion, and
mode-matching method. The radiated field was represented in a rapidly convergent series that is amenable to numerical analysis. Numerical results were presented to illustrate the radiation behaviors in terms of the corrugation geometries and compared with measurement data. The proposed theoretical approach is useful to estimate the radiation characteristic of the annular slot surrounded with surface corrugations and also finds its applications in the design of slot antennas.

**Appendix A**

The power orthogonality is given by the operation \( \langle \vec{p} | \vec{q} \rangle \) as

\[
\langle \vec{p} | \vec{q} \rangle_A^B = \frac{1}{2\pi} \int_0^{2\pi} \int_A^B [\vec{p} \times \vec{q}^*] \cdot \hat{z} \, d\rho \, d\phi, \quad A < \rho < B,
\]

(A1)

where the superscript * indicates the complex conjugate,

\[
\langle \vec{e}^l_{\text{tem}} | \vec{h}^l_{\text{tem}} \rangle_{a}^b = -\sqrt{\frac{\epsilon_2}{\mu_0}} \ln \frac{a}{b}, \quad (A2)
\]

\[
\langle \vec{e}^l_{\text{im}} | \vec{h}^l_{\text{im}} \rangle_{a}^b = 0, \quad (A3)
\]

\[
\langle \vec{e}^c_{\text{tem}} | \vec{h}^c_{\text{tem}} \rangle_{a}^b = -i \omega \epsilon_2 \Phi_0 \left( \gamma_{0q_2}^{(2)} b, c \right), \quad (A4)
\]

\[
\langle \vec{e}^l_{\text{im}} | \vec{h}^l_{\text{im}} \rangle_{a}^b = \frac{2\omega \epsilon_2 \Phi_0^{(1)} \gamma_{0q_2}^{(2)} \Phi_0 \left( \gamma_{0q_2}^{(2)} b, c \right)}{\pi \left( \gamma_{0n_1}^{(1)} - \gamma_{0q_2}^{(2)} \right) N_0 \left( \gamma_{0n_1}^{(1)} b \right)}, \quad (A5)
\]

\[
\langle \vec{e}^c_{\text{tem}} | \vec{h}^c_{\text{tem}} \rangle_{a}^c = -\sqrt{\frac{\epsilon_2}{\mu_0}} \ln \frac{a}{c}, \quad (A6)
\]

\[
\langle \vec{e}^l_{\text{im}} | \vec{h}^l_{\text{im}} \rangle_{a}^c = 0, \quad (A7)
\]

\[
\langle \vec{e}^c_{\text{tem}} | \vec{h}^c_{\text{tem}} \rangle_{a}^c = 0, \quad (A8)
\]

\[
\langle \vec{e}^l_{\text{im}} | \vec{h}^l_{\text{im}} \rangle_{a}^c = \frac{2\omega \epsilon_2 \Phi_0^{(1)} \gamma_{0q_2}^{(2)} \left( N_0 \left( \gamma_{0q_2}^{(2)} \right)^2 \right)^{\frac{1}{2}}}{\pi^2}, \quad (A9)
\]

\[
\langle \vec{e}^l_{\text{im}} | \vec{h}^l_{\text{im}} \rangle_{0}^\infty = \omega \epsilon_s \kappa_s \zeta, \quad (A10)
\]

\[
\langle \vec{e}^c_{\text{tem}} | \vec{h}^c_{\text{tem}} \rangle_{a_0}^{b_0} = -i \omega \epsilon_5 [J_0(\zeta)]_{z=a_0}^{b_0}. \quad (A11)
\]
The integrals in each simultaneous equations (47), (48), (50), and (51) are as follows:

\[
\left( \tilde{\zeta}_{t}(t) \right)_{a_{0}}^{b_{0}} = -k_{0}^{(3)} \frac{2\omega \varepsilon_{5}}{\pi} \frac{\zeta^{2}}{\xi^{2} - \gamma_{03}^{(3)^{2}}} \left[ \frac{J_{m}(\zeta)}{N_{0} \left( \gamma_{03}^{(3)} \right)} \right]_{z=a_{0}}^{b_{0}},
\]  
(A12)

\[
\left( \tilde{\zeta}_{m}(t) \right)_{a_{j}}^{b_{j}} = -i\omega \varepsilon_{5} [J_{0}(\zeta)]_{z=a_{j}}^{b_{j}},
\]  
(A13)

\[
\left( \tilde{\zeta}_{m}(t) \right)_{a_{j}}^{b_{j}} = -k_{0}^{(4)(j)} \frac{2\omega \varepsilon_{5}}{\pi} \frac{\zeta^{2}}{\xi^{2} - \gamma_{04}^{(4)(j)^{2}}} \left[ \frac{J_{m}(\zeta)}{N_{0} \left( \gamma_{04}^{(4)(j)} \right)} \right]_{z=a_{j}}^{b_{j}}.
\]  
(A14)

Similarly, the other power orthogonality can be obtained by setting the integration interval.

Appendix B

The integrals in each simultaneous equations (47), (48), (50), and (51) are as follows:

\[
I_{1} = \int_{0}^{\infty} [J_{0}(\xi) b_{0} - J_{0}(\xi a_{0})]^{2} \frac{1}{\kappa_{5}^{3}} d\xi,
\]  
(B1)

\[
I_{2} = -\int_{0}^{\infty} \left[ \frac{J_{0}(\xi)}{N_{0} \left( \gamma_{03}^{(3)} \right)} \right]_{z=a_{0}}^{b_{0}} [J_{0}(\xi)]_{z=a_{0}}^{b_{0}} \frac{\xi}{\kappa_{5}^{3}} \left( \xi^{2} - \gamma_{03}^{(3)^{2}} \right) d\xi,
\]  
(B2)

\[
I_{3} = \int_{0}^{\infty} [J_{0}(\xi)]_{z=a_{j}}^{b_{j}} [J_{0}(\xi)]_{z=a_{0}}^{b_{0}} \frac{1}{\kappa_{5}^{3}} d\xi,
\]  
(B3)

\[
I_{4} = -\int_{0}^{\infty} \left[ \frac{J_{0}(\xi)}{N_{0} \left( \gamma_{04}^{(4)(j)} \right)} \right]_{z=a_{j}}^{b_{j}} [J_{0}(\xi)]_{z=a_{0}}^{b_{0}} \frac{\xi}{\kappa_{5}^{3}} \left( \xi^{2} - \gamma_{04}^{(4)(j)^{2}} \right) d\xi,
\]  
(B4)

\[
I_{5} = -\int_{0}^{\infty} [J_{0}(\xi)]_{z=a_{0}}^{b_{0}} \left[ \frac{J_{0}(\xi)}{N_{0} \left( \gamma_{03}^{(3)} \right)} \right]_{z=a_{0}}^{b_{0}} \frac{\xi}{\kappa_{5}^{3}} \left( \xi^{2} - \gamma_{03}^{(3)^{2}} \right) d\xi,
\]  
(B5)

\[
I_{6} = \int_{0}^{\infty} \left[ \frac{J_{0}(\xi)}{N_{0} \left( \gamma_{03}^{(3)} \right)} \right]_{z=a_{0}}^{b_{0}} \left[ \frac{J_{0}(\xi)}{N_{0} \left( \gamma_{03}^{(3)} \right)} \right]_{z=a_{0}}^{b_{0}} \frac{\xi^{3}}{\kappa_{5}^{3}} \left( \xi^{2} - \gamma_{03}^{(3)^{2}} \right) \left( \xi^{2} - \gamma_{03}^{(3)^{2}} \right) d\xi,
\]  
(B6)

\[
I_{7} = -\int_{0}^{\infty} [J_{0}(\xi)]_{z=a_{j}}^{b_{j}} \left[ \frac{J_{0}(\xi)}{N_{0} \left( \gamma_{03}^{(3)} \right)} \right]_{z=a_{0}}^{b_{0}} \frac{\xi}{\kappa_{5}^{3}} \left( \xi^{2} - \gamma_{03}^{(3)^{2}} \right) d\xi,
\]  
(B7)
I_8 = \int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n4}^{(4)/(j)} z)} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q4}^{(4)/(j)} z)} \right]^{b_i} \frac{\zeta^3}{\kappa_5 \left( \zeta^2 - \gamma_{0n4}^{(4)/(j)} \right) \left( \zeta^2 - \gamma_{0q4}^{(4)/(j)} \right)} d\zeta, \tag{B8}

I_9 = \int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n3}^{(3)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q3}^{(3)}} \right]^{b_i} \frac{1}{\kappa_5 \zeta} d\zeta, \tag{B9}

I_{10} = -\int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n3}^{(3)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q3}^{(3)}} \right]^{b_i} \frac{\zeta}{\kappa_5 \left( \zeta^2 - \gamma_{0n3}^{(3)} \right)^2} d\zeta, \tag{B10}

I_{11} = \int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n3}^{(3)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q3}^{(3)}} \right]^{b_i} \frac{1}{\kappa_5 \zeta} d\zeta, \tag{B11}

I_{12} = -\int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n3}^{(3)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q3}^{(3)}} \right]^{b_i} \frac{\zeta}{\kappa_5 \left( \zeta^2 - \gamma_{0n3}^{(3)} \right)^2} d\zeta, \tag{B12}

I_{13} = -\int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n3}^{(3)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q4}^{(4)}} \right]^{b_i} \frac{\zeta}{\kappa_5 \left( \zeta^2 - \gamma_{0n3}^{(3)} \right) \left( \zeta^2 - \gamma_{0q4}^{(4)} \right)} d\zeta, \tag{B13}

I_{14} = \int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n3}^{(3)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q4}^{(4)}} \right]^{b_i} \frac{\zeta^3}{\kappa_5 \left( \zeta^2 - \gamma_{0n3}^{(3)} \right) \left( \zeta^2 - \gamma_{0q4}^{(4)} \right)} d\zeta, \tag{B14}

I_{15} = -\int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n4}^{(4)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q4}^{(4)}} \right]^{b_i} \frac{\zeta}{\kappa_5 \left( \zeta^2 - \gamma_{0n4}^{(4)} \right)^2} d\zeta, \tag{B15}

I_{16} = \int_0^\infty \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0n4}^{(4)}} \right]^{b_j} \left[ \frac{J_0(\zeta z)}{N_0(\gamma_{0q4}^{(4)}} \right]^{b_i} \frac{\zeta^3}{\kappa_5 \left( \zeta^2 - \gamma_{0n4}^{(4)} \right) \left( \zeta^2 - \gamma_{0q4}^{(4)} \right)} d\zeta. \tag{B16}

These integrals in Eqs. (B1) through (B16) can be calculated by using Gaussian quadrature integration.

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