Enhanced electromagnetic transmission through a slit surrounded by rectangular grooves

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Enhanced electromagnetic transmission through a slit surrounded by rectangular grooves

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Electromagnetic transmission through a slit surrounded by rectangular grooves in a conducting plane is investigated. An electromagnetic boundary-value problem associated with a slit surrounded by rectangular grooves in a conducting plane is rigorously solved based on the Fourier transform, eigenfunction expansion and mode matching method. The transmission coefficient through the slit is represented in a series. Computation is performed to illustrate the effect of the groove geometry on the transmission behaviours.

**Keywords:** electromagnetic transmission; slit; groove; surface plasmon polariton; mode matching method

1. Introduction

Electromagnetic scattering from slits in a conducting plane is a canonical problem in diffraction theory. Recently, there have been extensive studies on the enhanced optical transmission through subwavelength slits surrounded by surface corrugations associated with the surface plasmon polariton (SPP) resonances and slit waveguide modes (or Fabry–Perot-like modes) (Garcia-Vidal, Lezec, Ebbesen, & Martin-Moreno, 2003; Garcia-Vidal & Martin-Moreno, 2002; Takakura, 2001). The enhanced microwave and millimetrewave transmissions through subwavelength slits in corrugated plate have been also investigated experimentally and numerically (Beruete et al., 2004; Hibbins, Sambles, & Lawrence, 2002; Sutinjo & Okoniewski, 2012; Yang & Sambles, 2002). These enhanced transmissions in microwave and millimetrewave regimes are due to SPP-like modes, which are bound electromagnetic surface waves mimicking SPP with structured conducting surfaces (Pendry, Martin-Moreno, & Garcia-Vidal, 2004) and slit waveguide modes (Yang & Sambles, 2002). However, previous analytical studies have considered only the fundamental modes in slits and corrugations or the periodic corrugations or grooves, but the full-wave analysis including higher-order modes for the nonperiodic corrugations or grooves has not been presented. Therefore, it is of interest to rigorously solve the scattering from a slit surrounded by a finite number of corrugations or grooves in a conducting plane. In this paper, we shall solve the electromagnetic boundary-value problem of a slit surrounded by rectangular grooves in a conducting plane using the Fourier transform, eigenfunction expansion and mode matching method. The transmission coefficients are represented in a series and computed in terms of the groove geometry to illustrate the transmission behaviours.

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2. Field representations
Assume that a transverse magnetic (TM) polarised plane wave impinges on a slit surrounded with $2N$ rectangular grooves in a conducting plane, as shown in Figure 1. The dielectric constants of regions I, II, III and IV are $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, and $\varepsilon_4$, respectively. A time convention of $e^{-j\omega t}$ is suppressed throughout the analysis. In region I ($z > 0$), the total electromagnetic field consists of the incident, reflected and scattered components. The incident and reflected components are

$$H_{\text{it}}^i(x, z) = e^{jk_x x + jk_z z}$$

$$E_{\text{rt}}^i(x, z) = \mp \frac{k_z}{j \omega \varepsilon_1} e^{jk_x x + jk_z z}$$

where $k_x = k_1 \sin \theta_{\text{in}}$, $k_z = k_1 \cos \theta_{\text{in}}$, and $k_j = \omega \sqrt{\mu_j \varepsilon_j}$. Let us represent the scattered components based on the Fourier transform as

$$H_y^s(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{H}_y^s(\zeta) e^{-j\kappa_1 x + jk_1 z} d\zeta$$

$$E_x^s(x, z) = \frac{1}{\omega \varepsilon_1} \int_{-\infty}^{\infty} \frac{\kappa_1}{2\pi} \tilde{H}_y^s(\zeta) e^{-j\kappa_1 x + jk_1 z} d\zeta$$

where $\kappa_1 = \sqrt{k_j^2 - \zeta^2}$. In region II ($IT - w < x < IT + w, -d < z < 0$), the electromagnetic field takes the form of

$$H_y^{\text{III}}(x, z) = \sum_{m=0}^{\infty} A_m^{(I)} \cos[k_m(z + d)] \cos[a_m(x + w - IT)]$$

Figure 1. Problem geometry.
\[ E_x(x, z) = \frac{1}{j\omega \varepsilon_3} \sum_{m=0}^{\infty} A_m^{(l)}(-\chi_m) \sin[\chi_m(z + d)] \cos[a_m(x + w - IT)] \]  \hspace{1cm} (6)

where \( l = -N, ..., -1, 1, ..., N \), \( a_m = m\pi/(2a) \) and \( \chi_m = \sqrt{k_2^2 - a_m^2}. \) In region III \((-a < x < a, -h < z < 0)\), the electromagnetic field is

\[ H_y^{(II)}(x, z) = \sum_{m=0}^{\infty} [B_m \cos(\xi_m z) + C_m \sin(\xi_m z)] \cos[b_m(x + a)] \]  \hspace{1cm} (7)

\[ E_x^{(II)}(x, z) = \frac{1}{j\omega \varepsilon_3} \sum_{m=0}^{\infty} \xi_m [B_m \sin(\xi_m z) + C_m \cos(\xi_m z)] \cos[b_m(x + a)] \]  \hspace{1cm} (8)

where \( b_m = m\pi/(2a) \) and \( \xi_m = \sqrt{k_3^2 - b_m^2}. \) In region IV \((z < -h)\), we express the electromagnetic field based on the Fourier transform as

\[ H_y^{(IV)}(x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{H}_y^{(IV)}(\zeta) e^{-j\zeta x - j\zeta(z+h)} d\zeta \]  \hspace{1cm} (9)

\[ E_x^{(IV)}(x, z) = \left( \frac{-1}{\omega \varepsilon_4} \right) \int_{-\infty}^{\infty} \frac{\kappa_4}{2\pi} \tilde{H}_y^{(IV)}(\zeta) e^{-j\zeta x - j\kappa_4(z+h)} d\zeta \]  \hspace{1cm} (10)

where \( \kappa_4 = \sqrt{k_4^2 - \zeta^2}. \)

3. Enforcement of boundary conditions

To obtain the simultaneous equations for modal coefficients \( A_m^{(l)}(l = -N, ..., -1, 1, ..., N) \), \( B_m \) and \( C_m \), we enforce the boundary conditions at \( z = 0 \) and \( z = -h \). The tangential electric field continuity at \( z = 0 \) and \( z = -h \) require, respectively,

\[ E_x^l(x, 0) + E_x^s(x, 0) + E_x^c(x, 0) = \begin{cases} E_x^{(II)}(x, 0), & \text{on the } l\text{th groove} \\ E_x^{(III)}(x, 0), & \text{on the slit} \\ 0, & \text{elsewhere} \end{cases} \]  \hspace{1cm} (11)

\[ E_x^{(IV)}(x, -h) = \begin{cases} E_x^{(III)}(x, -h), & \text{on the slit} \\ 0, & \text{elsewhere}. \end{cases} \]  \hspace{1cm} (12)

Taking the Fourier transform to Equations (11) and (12) yields, respectively,

\[ \tilde{H}_y^l(\zeta) = \frac{\varepsilon_1}{\varepsilon_2} \sum_{l=-N}^{N} \sum_{m=0}^{\infty} A_m^{(l)}(-\chi_m) \sin(\chi_m(d + \varepsilon_3)) \frac{\zeta}{\kappa_1} e^{j\zeta IT} F_m^c(w) \]  \hspace{1cm} (13)

\[ + \frac{\varepsilon_1}{\varepsilon_3} \sum_{m=0}^{\infty} \xi_m C_m \frac{\zeta}{\kappa_3} F_m^c(a) \]

\[ \tilde{H}_y^{(IV)}(\zeta) = \frac{\varepsilon_4}{\varepsilon_2} \sum_{m=0}^{\infty} (-\xi_m)[B_m \sin(\xi_m h) + C_m \cos(\xi_m h)] \frac{\zeta}{\kappa_4} F_m^c(a) \]  \hspace{1cm} (14)
where

\[ F_p^x(y) = \frac{(-1)^p e^{jxy} - e^{-jxy}}{(p \pi/(2y))^2 - x^2}. \]  

(15)

The continuities of tangential magnetic field at \( z = 0 \) are

\[ H_x^j(x, 0) + H_y^j(x, 0) + H_y^a(x, 0) = \begin{cases} H_x^{(1)}(x, 0), & \text{on the } jth \text{ groove} \\ H_x^{(11)}(x, 0), & \text{on the slit.} \end{cases} \]  

(16)

Substituting \( H_y^s(\zeta) \) into Equation (15) and applying the orthogonality of eigenfunctions to Equation (15) gives

\[ j2k_e e^{k_e z} F_n^{k_e}(w) = \frac{j \varepsilon}{2 \pi \varepsilon} \sum_{l=-N}^{N} \sum_{m=0}^{\infty} A_m^{(l)} (-\chi_m) \sin(\chi_m d) I_{l_{m,n}}^{1}(k_1, T, w, w) \]

\[ + \frac{j \varepsilon}{2 \pi \varepsilon} \sum_{m=0}^{\infty} \xi_m C_m^{0l_m} (k_1, T, a, w) + A_m^{(l)} a_n \cos(\chi_n d) w \]  

(17)

\[ j2k_e F_n^{k_e}(a) = \frac{j \varepsilon}{2 \pi \varepsilon} \sum_{l=-N}^{N} \sum_{m=0}^{\infty} A_m^{(l)} (-\chi_m) \sin(\chi_m d) I_{l_{m,n}}^{10}(k_1, T, w, a) \]

\[ + \frac{j \varepsilon}{2 \pi \varepsilon} \sum_{m=0}^{\infty} \xi_m C_m^{0l_m} (k_1, T, a, a) + a_n B a \]  

(18)

where \( a_0 = 2, a_n = 1(n = 1, 2, 3, \ldots) \) and

\[ I_{l_{m,n}}^{1}(k_1, T, w, w) = \int_{-\infty}^{\infty} \frac{\xi^2}{k_1} e^{(l-1)T} F_n^{c}(w) F_m^{c}(-w) d\zeta \]  

(19)

\[ I_{l_{m,n}}^{0}(k_1, T, a, w) = \int_{-\infty}^{\infty} \frac{\xi^2}{k_1} e^{(l-r)T} F_n^{c}(a) F_m^{c}(-w) d\zeta \]  

(20)

\[ I_{l_{m,n}}^{10}(k_1, T, w, a) = \int_{-\infty}^{\infty} \frac{\xi^2}{k_1} e^{(l)T} F_n^{c}(w) F_m^{c}(-a) d\zeta \]  

(21)

\[ I_{l_{m,n}}^{00}(k_1, T, a, a) = \int_{-\infty}^{\infty} \frac{\xi^2}{k_1} F_n^{c}(a) F_m^{c}(-a) d\zeta. \]  

(22)

Using the residue integral technique (Lee, Eom, Cho, & Chun, 1996), we can calculate Equations (18), (19), (20), and (21). Similarly, the continuity of tangential magnetic field at \( z = -h \) yields
\[ [B_n \cos(\xi_n h) - C_n \sin(\xi_n h)] \alpha_n a = \frac{j \varepsilon_4}{2\pi \varepsilon_3} \sum_{m=0}^{\infty} \xi_m [B_m \sin(\xi_m h) + C_m \cos(\xi_m h)] \cdot I_{m,0}^{0,0}(k_4, T, a, a). \]  

(23)

It is possible to solve a set of the simultaneous Equations (16), (17) and (22) for the modal coefficient \( A_m^l (l = -N, ..., -1, 1, ..., N) \), \( B_m \) and \( C_m \).

4. Numerical results

Transmission coefficient \( \tau \) is defined as a ratio of the time-averaged power transmitted through the slit to the time-averaged power incident on the slit as follows

\[
\tau = -3 \left\{ \frac{\varepsilon_1}{2k_1 \varepsilon_3} \sum_{m=0}^{\infty} a_m \xi_m \left[ |B_m|^2 \sin(\xi_m h) \{ \cos(\xi_m h) \}^* - B_m C_m^* \sin(\xi_m h) |^2 \right. \\
+ \left. C_m B_m^* \cos(\xi_m h) |^2 - |C_m| \cos(\xi_m h) \{ \sin(\xi_m h) \}^* \right] \right\}.
\]

(24)

where the superscript * indicates the complex conjugate. To check the validity of our formulation, we plot the normalised transmission coefficient versus frequency for different incident angles as shown in Figure 2. The number of modes used in the computation is \( m = 3 \) including propagating modes and a few evanescent modes to achieve convergence. The comparison between our result (\( \theta_m = 0^\circ \)) and measure data (Hibbins et al., 2002) generally shows a good agreement. It is seen that the transmission is enhanced around 46 GHz due to the slit waveguide modes. As the incident angle increases, the transmission coefficient decreases due to the mismatch between slit waveguide modes and incident fields. Figure 3 shows the transmission coefficient versus both frequency and groove width. It is interesting to note that the transmission becomes maximum when the groove width has an optimum value. This is because the magnitude of SPP-like mode becomes maximum when the groove width has an optimum value, which can be estimated by using Floquet's theorem under the assumption of periodic groove array. The reflectance of the periodic groove array is given in (Lopez-Rios, Mendoza, Garcia-Vidal, Sanchez-Dehesa, & Pannetier, 1998) as

\[
R = 1 + \frac{i 4w k \sin^2(k_x w)}{T} \frac{1}{D}.
\]

(25)

where

\[
D = \cot(kd) - i \frac{2w}{T} \sum_{n=-\infty}^{\infty} \frac{k \sin^2(k_{x,n} w)}{k_{x,n}},
\]

(26)

where \( k_{x,n} = k_x + 2\pi n / T \) and \( k_{x,n} = \sqrt{k^2 - k_{x,n}^2} \). For example, the transmission coefficient is maximum at \( 2w = 1.2 \) mm and \( f = 13.8 \) GHz in Figure 3. We can show that the reflectance is minimum under the same conditions using Equation (24). This is because the magnitude of SPP-like mode becomes maximum when the reflectance from the groove array is minimum. Note that it was shown in (Garcia-Vidal et al., 2003) that the maximum enhanced transmission of these structures is achieved when the parameters except the groove width are chosen as
follows: $T \approx \lambda$, $h \approx n\lambda/2$ and $d \approx (2n + 1)\lambda/4$. Figure 4 illustrates the transmission coefficient versus frequency for different numbers of grooves. As the number of grooves increases, the transmission around 13.8 GHz increases due to enhancements in the magnitude of SPP-
like modes. It is seen that the transmission around 13.8 GHz becomes saturated as the number of grooves increases since this structure approaches to the periodic one. Also, the enhanced transmission due to silt waveguide modes can be observed around 8.8 GHz and 18.5 GHz including the case without grooves.

Figure 4. Transmission coefficient versus frequency ($2w = 1.2$ mm, $d = 4$ mm, $T = 20$ mm, $2a = 1.6$ mm, $h = 14$ mm, $\theta_m = 0^\circ$).

Figure 5. Transmission coefficient versus frequency ($2w = 1.2$ mm, $d = 4$ mm, $T = 20$ mm, $2a = 1.6$ mm, $h = 14$ mm, $2N = 10$, $\theta_m = 0^\circ$).
Figure 5 shows the transmission coefficient versus frequency for different groove distributions where total number of grooves is fixed \(2N = 10\). As the groove distribution becomes symmetric, the transmission coefficient increases around 13.8 GHz due to enhancements in the magnitude of SPP-like modes.

5. Conclusion

The electromagnetic boundary-value problem dealing with a slit surrounded by rectangular grooves in a conducting plane has been rigorously solved by using the Fourier transform, eigenfunction expansion and mode matching method. Transmission coefficient was represented in a series. Computation was performed to illustrate the enhanced transmission behaviours associated with SPP-like modes and slit waveguide modes in terms of the groove geometry. Also, the effect of the groove width on the transmission was discussed. Our theoretical formulation is useful to estimate the enhanced transmission through apertures surrounded by surface corrugations in a conducting plane and also finds its practical applications in aperture antennas and frequency selective surfaces.

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References


