and positive for all other \( n_\alpha \) and positive elsewhere, consistent with the requirement of the constitutive parameters of the dielectric-magnetic half-space: For the sake of illustration, the following values were selected for

\[ E_r(z, \lambda_0) = \int_{-\infty}^{\infty} E_r(z, t) \exp \left(-\frac{2\pi c_0}{\lambda_0} t \right) dt \] (11)

was calculated to determine the spectral contents of the incident and the reflected pulses. The parameters \( z, t_0, \) and \( t_b \) were chosen to capture as much of both pulses as possible, following the procedure described elsewhere [11]. The computed ratio of the Fourier transform of the reflected pulse to that of the incident pulse is denoted here by \( r_{\text{FD}}(\lambda_0) \), the subscript TD indicating its emergence from time-domain analysis.

3. NUMERICAL RESULTS AND DISCUSSION

For the sake of illustration, the following values were selected for the constitutive parameters of the dielectric-magnetic half-space:

\[ p_e = 1, p_m = 0.8, \lambda_e = 300 \text{ nm}, \lambda_m = 320 \text{ nm}, M_e = M_m = 100. \]

Thus, \( \varepsilon_r(\lambda_0) > 0 \) and \( \mu_r(\lambda_0) > 0 \) for all \( \lambda_0 \). However, \( \varepsilon_r(\lambda_0) \) is negative for \( \lambda_0 \in [212.1, 300] \text{ nm} \), but it is positive for all other \( \lambda_0 \); whereas \( \mu_r(\lambda_0) \) is negative for \( \lambda_0 \in [238.6, 320] \text{ nm} \), and positive for all other \( \lambda_0 \).

The definition of the refractive index in (2) suggests two possibilities: Either (a) \( n^*(\lambda_0) \) is negative for \( \lambda_0 \in [236.1, 316.8] \text{ nm} \) and positive elsewhere, consistent with the requirement of \( n^*(\lambda_0) > 0, \forall \lambda_0 \); or (b) \( n^*(\lambda_0) \) is negative for \( \lambda_0 \in [236.1, 316.8] \text{ nm} \) and positive elsewhere, consistent with the requirement of \( n^*(\lambda_0) \approx 0, \forall \lambda_0 \).

Thus, attention had to be focused on the anomalous spectral regime \( \lambda_0 \approx [236.1, 316.8] \text{ nm} \). In this regime, the reflection coefficient \( r(\lambda_0) \) for Possibility A is the reciprocal of that for Possibility B. The two possibilities can therefore be unambiguously distinguished from one another.

The carrier wavelength was chosen as \( \lambda_{\text{carrier}} = 240 \text{ nm} \). The pulse duration is 3 fs and its 3-dB band is \( \lambda_0 \in [218, 261] \text{ nm} \). Therefore the anomalous spectral regime was substantively covered by the time-domain calculations. The segment sizes used, \( \Delta z = 5 \text{ nm} \) and \( \Delta t = 0.015 \text{ fs} \), were adequate for the chosen constitutive parameters, but obviously would be totally inadequate in the resonance bands of \( \varepsilon_r(\lambda_0) \) and \( \mu_r(\lambda_0) \).

Possibility A is clearly nonsensical. It implies transport of energy in the half-space \( z \geq z_r \) toward the interface \( z = z_e \). Not surprisingly, therefore, (2), (5), and (6) yielded \( |r(\lambda_0)| > 1 \) for all \( \lambda_0 \approx [236.1, 316.8] \text{ nm} \).

Figure 1 presents the computed values of \( |r(\lambda_0)| \) obtained from (2), (5), and (6) for Possibility B (i.e., when \( n^*(\lambda_0) \approx 0 \) is guaranteed for all \( \lambda_0 \)). The computed values of \( |r_{\text{FD}}(\lambda_0)| \) are also shown therein. The two sets of magnitudes compare very well for \( \lambda_0 \leq 290 \text{ nm} \). Examination of the refracted pulse also showed that it transported energy away from the interface \( z = z_r \), which corroborates the observations of Ziolkowski and Heyman [4].

Thus, time-domain analysis validates the conclusion that \( n(\lambda_0) \) must be selected in frequency-domain research in such a way that \( n^*(\lambda_0) \approx 0 \)—irrespective of the sign of \( n^*(\lambda_0) \).

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COUPLING THROUGH A FLANGED COAXIAL LINE ARRAY

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Received 10 December 2001

ABSTRACT: Coupling through a flanged coaxial line array is investigated. The Hankel transform and superposition principle are used to...
obtain the coupling parameters. The mutual coupling expression is shown to be identical with other results based on the Green’s-function approach. Computation and experiment are performed to illustrate the coupling behavior. © 2002 Wiley Periodicals, Inc. Microwave Opt Technol Lett 33: 467–470, 2002; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10353

Key words: coaxial line array; Hankel transform; mutual coupling

1. INTRODUCTION

Open-ended coaxial lines have been widely used in reflector antenna feeds, nondestructive measurements, and interstitial microwave applicators [1–8]. A single open-ended coaxial line has been investigated extensively [1–5]. Coupling through a flanged coaxial line array is an important subject matter in antenna feeds and electromagnetic interference and compatibility applications. Theoretical and experimental investigation into coupling through a flanged coaxial line array has been performed in [6] with the use of the Green’s-function approach. A simple solution for TEM mode coupling through flanged coaxial lines is given by (1) and (9) in [6]. The motivation of the present Letter is twofold. One is to rederive the theoretical expression in [6] for the coupling between the flanged coaxial lines by using a Hankel-transform approach. The other is to perform an experiment in order to investigate the coupling behavior for multiple coaxial apertures on a conducting flange. In the next section, the theoretical coupling expression will be derived by representing the field in terms of the Hankel transform and eigenmode expansions.

2. THEORETICAL DEVELOPMENT

Consider a coaxial line array consisting of finite N number of open-ended coaxial lines \((j = 1, 2, \ldots, N)\) on an infinite flange. Assume that an incident field, TEM mode, propagates along the \(j\)th coaxial line (see Figure 1). A time convention of \(e^{-i\omega t}\) is suppressed throughout the analysis. The total \(E\) field in Region I (\(z < 0\), \(a_j < r < b_j\)) consists of the incident and reflected components as

\[
\bar{E}_0^1(r_j, \phi_j, z) = \frac{E_0^1}{r_j} (V e^{i\beta z} + V e^{-i\beta z}),
\]

\[
\bar{H}_0^1(r_j, \phi_j, z) = \frac{\phi}{r_j} \sqrt{\frac{1}{\mu r_j}} (V e^{i\beta z} - V e^{-i\beta z}),
\]

where \(\beta_j = \frac{\omega}{\sqrt{\varepsilon_j \mu}}\). In Region I, the higher-order reflected components are ignored. In Region II \((0 < z)\), the \(E\) field is a sum of individual components based on the superposition principle:

\[
\bar{E}_1^1 = \sum_{j=1}^{N} \bar{E}_0^1(r_j, \phi_j, z),
\]

\[
\bar{H}_1^1 = \sum_{j=1}^{N} \bar{H}_0^1(r_j, \phi_j, z),
\]

where \(\kappa = \sqrt{k_0^2 - \xi^2}\) and \(k_0 = \omega \sqrt{\varepsilon_0 \mu}\).

\[
\bar{E}_m = \left[ \frac{\bar{r} j \kappa \xi J_m^* (\xi r)}{r} - \frac{\phi}{r_j} \frac{V}{J_m (\xi r)} \right] e^{im\phi},
\]

\[
\bar{H}_m = \left[ \frac{\bar{r} j \kappa \xi J_m (\xi r)}{r} + \phi j \kappa \xi J_m^* (\xi r) \right] e^{im\phi},
\]

\[
\bar{E}_0 = \left[ -\frac{\bar{r} j \kappa \xi J_1^* (\xi r)}{r} - \phi j \kappa \xi J_0 (\xi r) \right] e^{im\phi},
\]

\[
\bar{H}_0 = \left[ \frac{\bar{r} j \kappa \xi J_1 (\xi r)}{r} - \frac{\phi}{r_j} \frac{V}{J_0 (\xi r)} \right] e^{im\phi}.
\]

It is necessary to enforce the boundary conditions on the field continuities. The continuity of \(E\) field at \(a < r < b\) and \(z = 0\) gives

\[
E_m^1(r, \phi, 0) = \begin{cases} E_0(r, \phi, 0), & a_j < r < b_j, \\ 0, & \text{otherwise.} \end{cases} \quad \text{(11)}
\]

Applying the Hankel transform to (11) \(\int_0^1 (11) J_1 (\xi r) r \, dr\) yields, respectively,

\[
\tilde{\Phi}_m^{11} (\xi) = -i (V + \frac{J_1 (\xi b) - J_0 (\xi a)}{\xi}),
\]

\[
\tilde{\Phi}_0^{11} (\xi) = 0. \quad \text{(13)}
\]

The continuity of the tangential \(H\) field for \(a_j < r_j < b_j\) at \(z = 0\)

\[
\sum_{j=1}^{N} \bar{H}_0^1(r_j, \phi_j, 0) = \bar{H}_0^1(r_j, \phi_j, 0), \quad a_j < r_j < b_j \quad \text{(14)}
\]

is rewritten as

\[
\sum_{j=1}^{N} \frac{\phi}{r_j} \int_0^1 \tilde{\Phi}_m^{11} (\xi) \xi \omega \kappa J_0 (\xi r_j) \, d\xi = -\frac{\phi}{r_j} \sqrt{\frac{V}{\mu \pi}} r_j d\phi. \quad \text{(15)}
\]

Applying the power orthogonality property

\[
\left( \frac{1}{2\pi} \int_0^1 \int_0^1 \right) \times \frac{1}{r_j} dr_j d\phi
\]
where
displacement between adjacent coaxial lines
plane (300 mm was conducted with a 3
In order to investigate the theoretical result (18), an experiment
3. NUMERICAL COMPUTATIONS AND MEASUREMENTS
3 coaxial-line square array (a = 0.815 mm, b = 2.655
mm, l = 7.35 mm).

\[-\frac{1}{2\pi} \sum_{k=1}^{N} \int_{0}^{\infty} \Phi_{mk}(\xi) \xi \omega E_{0} \int_{0}^{2\pi} J_{l}(\xi r_{k}) \cos(\phi_{k} - \phi_{j}) \, dr_{k} \, d\phi_{k} \, d\xi \int_{0}^{2\pi} \frac{1}{\mu} \sqrt{\frac{V_{j}}{V_{k}}} \int_{0}^{2\pi} \frac{1}{\mu} \sin(\phi_{k} - \phi_{j}) \, dr_{j} \, d\phi_{j}.
\]

(16)

If one substitutes \( \Phi_{mk}(\xi) \) of (12) into (16) and uses Graf’s addition
theorem
\[ J_{l}(\xi r_{k}) e^{i\phi_{k}} = \sum_{\mu = -\infty}^{\infty} J_{\mu-\nu}(\xi r_{k}) J_{\nu}(\xi r_{k}) e^{i\phi_{k}}, \]

(17)

where \( l_{jk} \) is a distance between the centers of the \( j \)th and \( k \)th
coaxial lines, one gets
\[ \sum_{k=1}^{N} \left[ (l_{jk} + \delta_{jk} \Gamma) V_{j} \right] + \sum_{k=1}^{N} \left[ (l_{jk} - \delta_{jk} \Gamma) V_{k} \right] = 0, \]

(18)

where \( \delta_{jk} \) is the Kronecker delta and
\[ I_{jk} = \int_{0}^{\infty} \left[ J_{l}(\xi r_{k}) - J_{l}(\xi r_{k}) \right] \frac{J_{l}(\xi r_{k})}{\xi} d\xi, \]

(19)

\[ C_{j} = \frac{1}{\omega E} \sqrt{\frac{\omega}{\mu}} \sin(b_{j}/a_{j}). \]

(20)

Note that the final results (18) and (19) are equivalent to (1) and (9)
in [6]. It is trivial to solve a set of simultaneous equations (18) for
the unknown coefficients, \( V_{k} \).

4. CONCLUSION
The behavior of coupling through a flanged coaxial line array has
been investigated theoretically and experimentally. A simple
solution for the coupling given in [6] was derived again with the use
of the Hankel transform and superposition principle. An experi-
ment has been conducted with a 3 \( \times \) 3 coaxial-line square array,
and the measured data have shown a good agreement with the theoretical results. The final theoretical expression is not only easy to understand, but also useful to estimate the coupling through multiple open-ended coaxial lines on a flange.

ACKNOWLEDGMENT
The authors would like to thank Mr. K. B. Park and Dr. S. H. Min at the Satellite Technology Research Center of Korea Advanced Institute of Science and Technology for their assistance in sample preparation and measurements.

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